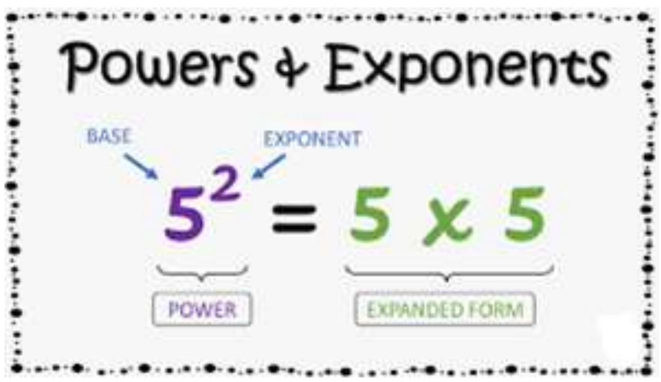
CLASS 8^{TH} MATHS

Exponents and Powers
Chapter -10

Exercise -10.1

Exponents and Powers

The concept of **Exponents and Powers** refers to a mathematical operation where a number (called the base) is raised to a certain power (called the exponent). It simplifies repeated multiplication of the same number.



1. Exponents

An exponent represents the number of times a base is multiplied by itself.

For example:

- 2^3 means $2 \times 2 \times 2 = 8$.
- Here, 2 is the base, and 3 is the exponent.

General Form: a^n

- a = Base.
- n = Exponent (or Power).

2. Laws of Exponents

These rules help simplify expressions involving exponents:

1. **Multiplication Rule:** $a^m \times a^n = a^{m+n}$
   Example: $2^3 \times 2^2 = 2^{3+2} = 2^5 = 32.$
2. **Division Rule:** $\frac{a^m}{a^n} = a^{m-n}$
   Example: $\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4.$
3. **Power of a Power:** $(a^m)^n = a^{m \cdot n}$
   Example: $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64.$
4. **Power of a Product:** $(ab)^n = a^n \cdot b^n$
   Example: $(2 \cdot 3)^2 = 2^2 \cdot 3^2 = 4 \cdot 9 = 36.$
5. **Power of a Quotient:** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
   Example: $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}.$
6. **Zero Exponent:** $a^0 = 1$ (for $a \neq 0$)
   Example: $5^0 = 1.$
7. **Negative Exponent:** $a^{-n} = \frac{1}{a^n}$
   Example: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}.$

3. Powers

Powers are an alternative way of representing large numbers in compact form.

Example of Large Numbers:

- $10^6 = 1,000,000.$
- $10^{-3} = 0.001.$

1. Evaluate.

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

Solution

(i) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $\left[a^{-n} = \frac{1}{a^n}\right]$

(ii) $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$ $\left[a^{-n} = \frac{1}{a^n}\right]$

(iii) $\left(\frac{1}{2}\right)^{-5} = (2)^5 = 32$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$

2. Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$ (iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$ (v) $2^{-3} \times (-7)^{-3}$

Solution

(*i*) $(-4)^5 \div (-4)^8$

$$= (-4)^{5-8} = (-4)^{-3} = \frac{1}{(-4)^3}$$

$$= \left(-\frac{1}{4}\right)^3 \qquad [\because \quad a^m \div a^n = a^{m-n}]$$

(*ii*) $\left(\frac{1}{2^3}\right)^2 = \frac{(1)^2}{(2^3)^2} = \frac{1}{2^6} = \left(\frac{1}{2}\right)^6$

Solution

When power is an even number and base is negative number then base number will become positive.

$$(iii)\ (-3)^4 \times \left(\frac{5}{3}\right)^4 = (-3)^4 \times \frac{(5)^4}{(3)^4}$$

$$= \frac{(3)^4 \times (5)^4}{(3)^4} = 5^4$$

$$(iv)\ (3^{-7} \div 3^{-10}) \times 3^{-5}$$

$$= 3^{-7-(-10)} \times 3^{-5}$$

$$= 3^{-7+10} \times 3^{-5}$$

$$= 3^3 \times 3^{-5} = 3^{3-5}$$

$$= 3^{-2} = \frac{1}{3^2} = \left(\frac{1}{3}\right)^2$$

$$(v)\ 2^{-3} \times (-7)^{-3}$$

$$= [2 \times (-7)]^{-3}$$

$$= (-14)^{-3} = -(14)^{-3}$$

$$= -\frac{1}{14^3} = \left(-\frac{1}{14}\right)^3$$

3. Find the value of.

(i) $(3^0 + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$ (v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

Solution

(i) $(3^0 + 4^{-1}) \times 2^2$

$$= \left(1 + \frac{1}{4}\right) \times 4$$

$$= \left(\frac{4+1}{4}\right) \times 4$$

$$= \frac{5}{4} \times 4 = 5$$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

$$= \left(\frac{1}{2} \times \frac{1}{4}\right) \div \frac{1}{2^2}$$

$$= \frac{1}{8} \div \frac{1}{4}$$

$$= \frac{1}{8} \times \frac{4}{1} = \frac{1}{2}$$

(iii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1}+4^{-1}+5^{-1})^0$ (v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

Solution

(iii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$

$$= 2^2 + 3^2 + 4^2$$

$$= 4 + 9 + 16 = 29$$

(iv) $(3^{-1}+4^{-1}+5^{-1})^0$

$$= \left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)^0$$

$$= \left(\frac{20+15+12}{60}\right)^0$$

$$= \left(\frac{47}{60}\right)^0 = 1 \qquad \left[\because \left(\frac{a}{b}\right)^0 = 1\right]$$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

$$= \left(-\frac{2}{3}\right)^{-2\times 2} = \left(-\frac{2}{3}\right)^{-4}$$

$$= \left(-\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}$$

4. Evaluate (i) $\frac{8^{-1} \times 5^3}{2^{-4}}$ (ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

Solution

(i) $\frac{8^{-1} \times 5^3}{2^{-4}}$

$$= \frac{\frac{1}{8} \times 125}{\frac{1}{2^4}} = \frac{125}{8} \times 2^4$$

$$= \frac{125}{\not{8}} \times \not{16}^{2} = 250$$

(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

$$= \left(\frac{1}{5} \times \frac{1}{2}\right) \times \frac{1}{6}$$

$$= \frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$$

5. Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Solution

$5^m \div 5^{-3} = 5^{-5}$

$\frac{5^m}{5^{-3}} = 5^5$

$\mathbf{5^m \times 5^3 = 5^5}$

$\mathbf{5^{m+3} = 5^5}$ $\quad (Using\ a^m \times a^n = a^{m+n})$

Comparing powers

m + 3 = 5

m = 5 – 3

m = 2

6. Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$

Solution

(i) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

$= (3-4)^{-1}$

$= (-1)^{-1} = \frac{1}{-1} = -1$

(ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$

$= \left(\frac{8}{5}\right)^{7}\times\left(\frac{8}{5}\right)^{-4} = \left(\frac{8}{5}\right)^{7-4}$

$= \left(\frac{8}{5}\right)^{3} = \frac{512}{125}$

7. Simplify.

(i) $\dfrac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$ (ii) $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Solution

$$\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$$

$$= \frac{25}{5^{-3} \times 10} \times \frac{t^{-4}}{t^{-8}}$$

$$= \frac{25}{5^{-3} \times 10} \times t^{-4} \times \frac{1}{t^{-8}}$$

$$= \frac{25}{5^{-3} \times 10} \times \frac{1}{t^{4}} \times \frac{t^{8}}{1}$$

$$= \frac{25}{5^{-3} \times 10} \times \frac{t^{8}}{t^{4}}$$

$$= \frac{25}{5^{-3} \times 10} \times t^{8-4}$$

$\left(Using\ \frac{a^m}{b^n} = a^{m-n}\right)$

$$= \frac{25}{5^{-3} \times 10} \times t^{4}$$

$$= \frac{25}{\left(\frac{1}{5^3}\right) \times 10} \times t^{4}$$

$$= \frac{25 \times 5^3}{10} \times t^{4}$$

$$= \frac{25 \times 5 \times 5 \times 5}{10} \times t^{4}$$

$$= \frac{25 \times 5 \times 5}{2} \times t^{4}$$

$$= \frac{625}{2}\ t^{4}$$

(ii) $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Solution

$= 3^{-5} \times 10^{-5} \times 125 \times \frac{1}{5^{-7}} \times \frac{1}{6^{-5}}$

$= \frac{1}{3^5} \times \frac{1}{10^5} \times 125 \times 5^7 \times 6^5$

$= \frac{6^5}{3^5} \times \frac{1}{10^5} \times 125 \times 5^7$

$= \left(\frac{6}{3}\right)^5 \times \frac{1}{10^5} \times 125 \times 5^7$ $\left(Using\ \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m\right)$

$= (2)^5 \times \frac{1}{10^5} \times 125 \times 5^7$

$= 2^5 \times \frac{1}{10^5} \times 5^3 \times 5^7$ (As $125 = 5 \times 5 \times 5 = 5^3$)

$= 2^5 \times \frac{1}{10^5} \times 5^{3+7}$ $\left(Using\ a^m \times a^n = (a)^{m+n}\right)$

$= 2^5 \times \frac{1}{10^5} \times 5^{10}$

$= \left(\frac{2}{10}\right)^5 \times 5^{10}$ $\left(Using\ \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m\right)$

$= \left(\frac{1}{5}\right)^5 \times 5^{10}$

$= \frac{1^5}{5^5} \times 5^{10}$

$= \frac{1}{5^5} \times 5^{10}$ $\left(Using\ \frac{a^m}{a^n} = a^{m-n}\right)$

$= \frac{5^{10}}{5^5}$

$= 5^{10-5}$

$= 5^5$